Section F – Algebra & Equations

Objective
In this section, we will be dealing with solving equations for the variable involved, manipulating formulae to get one variable as the subject of the formula and substitution of numbers into the equation for the variable.

Let us start with a quick revision of algebra and how to work with variables.
$x + x = 2x \neq 2 + x$

$x.x = x(x) = x \times x = x^2$

$\therefore xy = x \times y$

Now let us look at the simpler topic, namely *Substitution.*

For example: $y = aj + az - ag$, and the following information is given: a = 2, j = 3, z = 1 and g = 2.
The instruction is "Find the value of y."
All we need to do is substitute the values for a, j, z, and g into the given equation and calculate the answer.

REMEMBER: BODMAS APPLIES HERE. HENCE WE CALCULATE THE MULTIPLICATION FIRST, THEN ADDITION, THEN SUBTRACTION.

Therefore once we substitute the numbers in, we will have the following:
$\therefore y = 2(3) + 2(1) - 2(2) = 6 + 2 - 4 = 8 - 4 = 4$

The only thing to watch out for is that we substitute the right numbers in for the right variables and that we correctly apply BODMAS.

Equations

There are a few rules that must always be followed to solve an equation:

1. What we do to the left hand side (LHS) we must also do the right hand side (RHS).

2. When a number or variable moves to the other side of the equal sign, change the operation to the opposite operation.
   Addition ↔ Subtraction
   Multiplication ↔ Division

3. To solve for the variable, we must move all the terms with the variable in it to the one side of the equal sign and everything else must be moved to the other side.

If we take the example: $x+2=5$. We want to solve for *x*.
In words, this is saying "A number plus 2 will equal 5. What is the number?"

According to Rule 3, we want to keep all terms with x on one side and all the numbers on the other.
Thus we need to move the +2 to the other side. To do this we use Rule 2, which is to change the + 2 on the LHS into a – 2 on the RHS.

$$\therefore x = 5-2$$
$$\therefore x = 3$$

- To check that this answer is correct, we substitute our answer back into the **original equation** and check that LHS = RHS
- $$\therefore 3+2 = 5$$
  $$5 = 5$$
- We see that LHS = RHS, so our answer is correct.

Example: $3x-4=x+8$

- First step is to place all numbers containing x on the LHS and all the other numbers on the RHS. (**Rule 3**)
- To do this we need to move the -4 to the RHS and move the x to the LHS. When you move a number over the = signs you must change the sign. (**Rule 2**)
- Hence when you move the -4 over the equals sign to the RHS it will become a +4 and when you move the x over the equals sign to the LHS it will become $-x$.
- Therefore the formula will become: $3x - x = 8 + 4$
- Simplifying that, we get $2x = 12$
- To get x by itself, we need to move the 2 in front of x to the other side. (**Rule 3**)
- The 2 is being multiplied on the RHS, so when we move it, we must divide on the LHS. (**Rule 2**)
- Hence $x = {}^{12}/_{2} = 6$

⇒ More examples done = quicker the process becomes ⇐

If the instruction to us is to make a variable in the equation *the subject of the formula*, the instruction essentially is to change the equation so that it ends up in the form of that variable on one side with a coefficient of 1 and all the rest on the other side of the equal sign.

Example:

If we have the following example: $x = \sqrt{b^2 - 4ac}$

The instruction is to make c the subject of the equation.

$$\therefore x = \sqrt{b^2 - 4ac}$$

$$x^2 = b^2 - 4ac$$

$$x^2 - b^2 = -4ac$$

$$\frac{x^2 - b^2}{-4a} = c$$

This is our final answer since it has the variable we want with coefficient of 1 on the one side and everything else on the other side.

<u>Note:</u> If there is no number in front of the *x*, it is assumed that the number is 1.

<u>Activity</u>
Equations – Ex 4-16 pg 46-62
Changing the subject of the formula – Ex 4-9 pg 156-161

Section G – Number Sequences

Objective

This section will deal with strings of numbers that have rules that are used in order to get from each term in the string to the next. We will look at a few different types of questions.

Firstly, we look at additive rules. The best way to learn these are from examples.

Example 1: 1, 3, 5, 7, 9…
We can see that these are the set of odd numbers in our number system. The point that needs to be seen is that whichever number we start with, to get to the next one, we just add 2.

Example 2: 6, 3, 0, -3, -6…
The numbers here are decreasing. We see that each time, they decrease by 3, so from one number to the next, we subtract 3.

We look now at the multiplicative rules.

Example 3: 3, 12, 48, 192, 768…
The numbers are increasing here. We see that we are not adding or subtracting a constant amount each time, which means this is **not** an additive rule. We do see that to go from 3 to 12, we multiply by 4 and from 12 to 48, we multiply by 4.

Example 4: 36, 18, 9, 4.5, 2.25…
Again we are not adding/subtracting a constant amount. We see that we could be dividing by 2 since 36 divided by 2 is 18 and 18 divided by 2 gives 9.

Now since we know the rules about how to move from one number to the next, lets look at how we could get any "number" (properly known as a *term*) in the sequence. To do this, we use *general forms or the equation for the n^{th} term*.

Taking additive rule questions, the general term is in the form of $T_n = dn + b$ where T_n is any term in the sequence, n is the term number, d is the constant amount added/subtracted (as was discovered in the above examples) and b will be discovered with the table.

Sequence	n	dn = 2n (ie: d = 2)	dn+b = 2n-1 (ie: b = -1)
1	1	2	2-1=1
3	2	4	4-1=3
5	3	6	6-1=5
7	4	8	8-1=7
9	5	10	10-1=9

We saw from the Sequence column and the *dn* column that we needed to subtract 1 each time to get the same value as in the Sequence column. This is how we determine the b value.
The n column will always start with 1 and increase by 1 each time, since n represents the term number.

Let us look at how Example 2 will work.

Sequence	n	dn = 3n (ie: d = 3)	dn+b = -3n+9 (ie: b = 9)
6	1	-3	-3+9=6
3	2	-6	-6+9=3
0	3	-9	-9+9=0
-3	4	-12	-12+9=-3
-6	5	-15	-15+9=-6

We saw from the Sequence column and the dn column that we needed to add 9 each time to get the same value as in the Sequence column.

The general form for the multiplicative rule is different to the additive rule. The form is: $T_n = b.r^{n-1}$ where r is the amount being multiplied every time and b is found through using the table.

NOTE: THE EXPONENT/INDEX IS N-1, NOT JUST N!!

Sequence	n	$r^{n-1}=4^{n-1}$ (ie: r = 4)	$b.r^{n-1}=3.4^{n-1}$ (ie: b = 3)
3	1	1	3(1)=3
12	2	4	3(4)=12
48	3	16	3(16)=48
192	4	64	3(64)=192
768	5	256	3(256)=768

Try to do Example 4 now. The rule should come out as $36(½)^{n-1}$.

If they give you the general form, and ask for the 5^{th} term say, all that is needed is to substitute the *n* for 5 and calculate.

Example 6: $T_n = \dfrac{n^2 - 2}{n^2 + 2}$

We are asked for the 5^{th} and 20^{th} terms.

$$\therefore T_5 = \frac{5^2 - 2}{5^2 + 2} = \frac{23}{27}$$

$$\therefore T_{20} = \frac{20^2 - 2}{20^2 + 2} = \frac{398}{402}$$

NOTE: ANOTHER TYPE OF RULE IS WHEN THE ONE TERM PLUS THE NEXT TERM GIVES THE FOLLOWING TERM – WATCH OUT FOR THIS!!

Activity

Sequences – Ex 7-8 pg 7-8

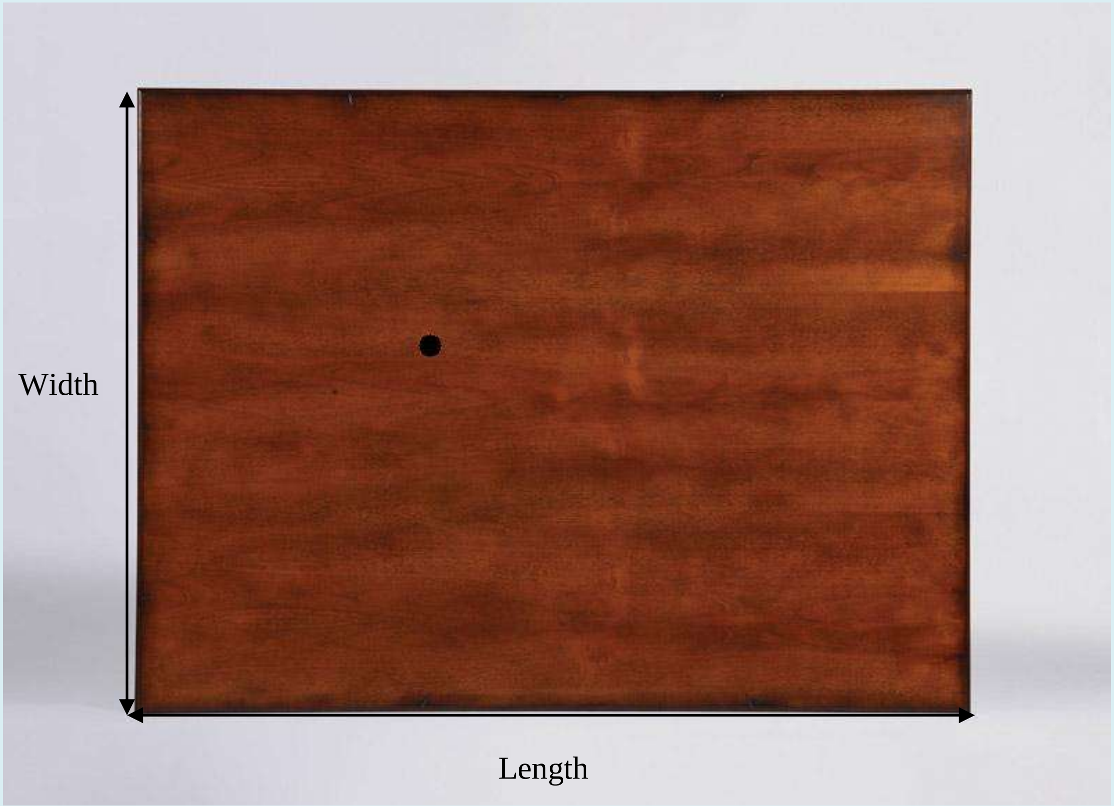
Section H - Cartesian plane & straight lines

Objective

This section will introduce to you what a Cartesian plane is, how to use it and how to analyse it. We will also be looking at straight lines. We will look at how to draw them from tables, how to analyse them from graphs and also the general equation.

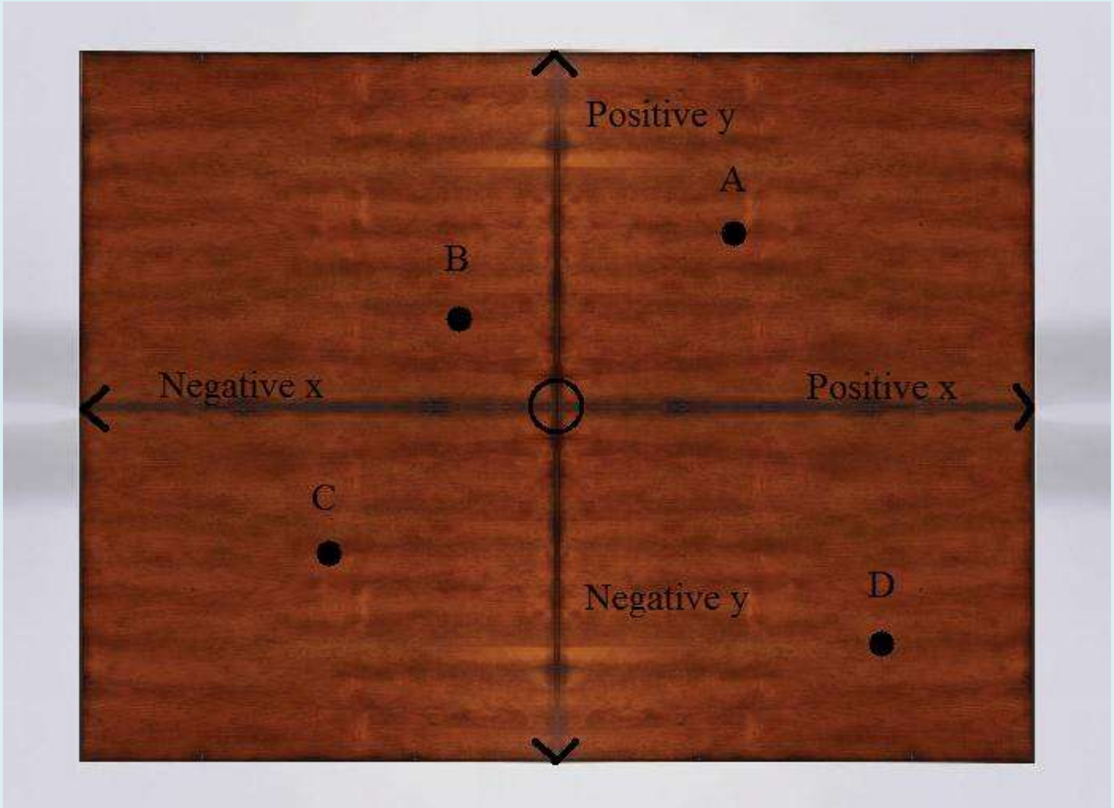
The Cartesian plane is very important in Maths. It will stay with you until the end of your school career.

It is based on the idea of different points on a flat table top.



If I want to describe where that black dot is on the table top, all I would need to tell is how high up the width and how far along the length it is from one of the corners. With those 2 pieces of information, I can specify a unique position on the table.

If we consider the Length value as an *x*-value and the Width value as a *y*-value, then we have *Cartesian co-ordinates*. If we put 4 of those tables together so that the starting "corner" is in the middle, then we get a Cartesian plane. We take that "corner" to be zero on both the *x*-axis (horizontal black line) and *y*-axis (vertical black line). Thus, to the left or upwards from zero is positive and to the right or downwards from zero is negative.
The following diagram demonstrates this:



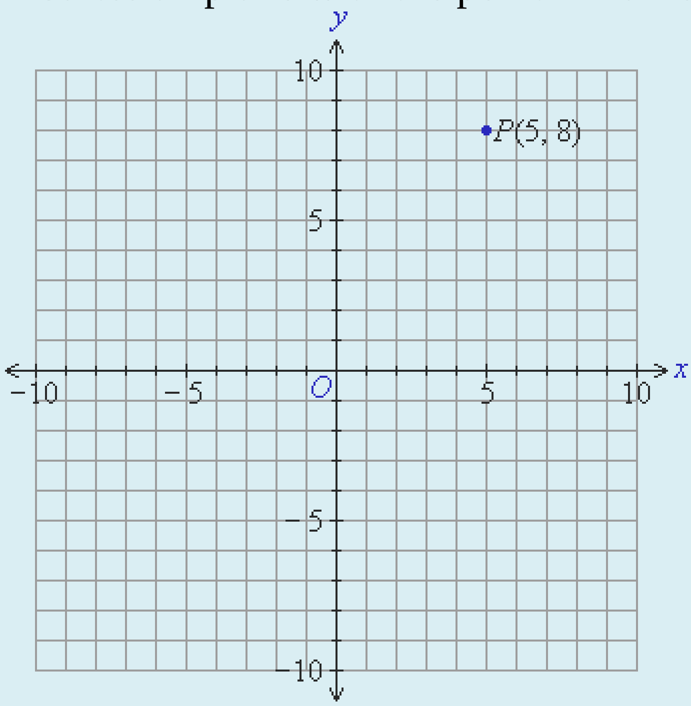
The circle at the centre shows us that that is the *Origin*, being the zero point.

We can see that the Cartesian plane is split into 4 sections. These sections are known as *Quadrants*. Below is a table that demonstrates how the co-ordinates would look like in each of the 4 quadrants.

Point A	(+ , +)
Point B	(- , +)
Point C	(- , -)

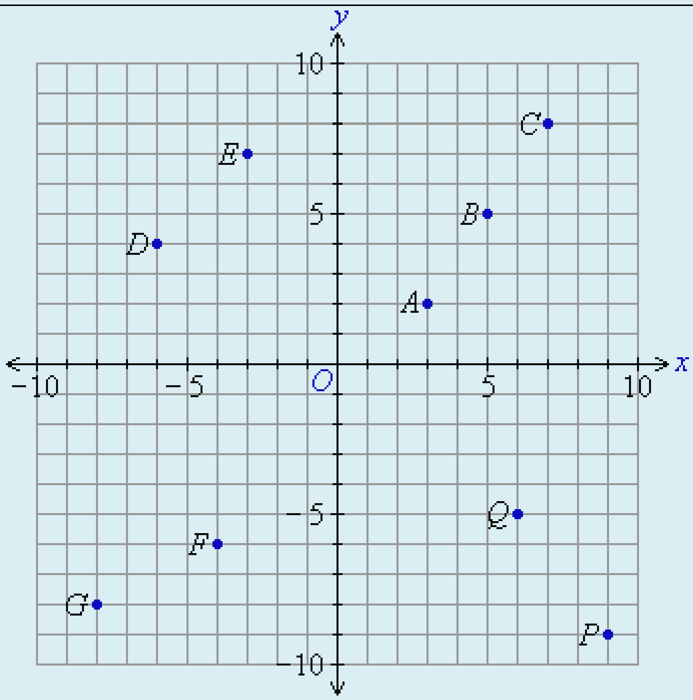
Point D	(+ , -)

This is an actual Cartesian plane with the point P marked on it.



When we write down the co-ordinates, we always write the *x*-value first and then the *y*-value, for example, (5;8). This says that we need to go 3 units to the right of the origin on the *x*-axis and then 8 units up from that point.

<u>Exercise:</u> Practice naming co-ordinates.



Try to write down the co-ordinates of each point, from A to Q, and then look at the answers to check how you have done.

Answers:

- *A* is 3 units to the right of and 2 units above the origin. So, point *A* is (3, 2).

- *B* is 5 units to the right of and 5 units above the origin. So, point *B* is (5, 5).

- *C* is 7 units to the right of and 8 units above the origin. So, point *C* is (7, 8).

- *D* is 6 units to the left of and 4 units above the origin. So, point *D* is (–6, 4).

- *E* is 3 units to the left of and 7 units above the origin. So, point *E* is (–3, 7).

- *F* is 4 units to the left of and 6 units below the origin. So, point *F* is (–4, –6).

- *G* is 8 units to the left of and 8 units below the origin. So, point *G* is (–8, –8).

- *P* is 9 units to the right of and 9 units below the origin. So, point *P* is (9, –9).

- *Q* is 6 units to the right of and 5 units below the origin. So, point *Q* is (6, –5).

Graphs

Just as with Sequences, there is a *General Form* equivalent with graphs. Here we need a relationship between x any y to create a list of co-ordinates. These co-ordinates will make a shape on the Cartesian plane. This is known as a *graph*.

Once given the relationship, a way to calculate the shape of the graph is to use a table.

- *How do I use a table?*
  Use the x-values that have been given (or choose some if they have not been given) by substituting them into the x in the equation and solve for the corresponding y-values.

- If the values that you choose do not give you enough information about the shape, choose MORE points!!

Example:

1) $y = 3x - 2$ for $0 \le x \le 6$.

x	0	1	2	3	4	5	6
$3x$	0	3	6	9	12	15	18

-2	-2	-2	-2	-2	-2	-2	-2
$y = 3x - 2$	-2	1	4	7	10	13	16

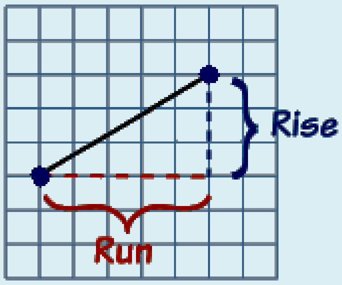
We can see that we now have 7 points with co-ordinates to plot on a Cartesian plane to see what graph it is.

It is a *straight line* when joined!!

- How is the gradient calculated?
  We need to 2 points with the exact co-ordinates to calculate the gradient. Once we have those, we can use the formula:

  $$gradient = \frac{rise}{run}$$

  Rise is the vertical difference between the 2 points and run is the horizontal difference.



Example:

We have the points (1,1) and (-1;-3) from above.

To calculate the vertical difference, we subtract the *y*-values from each other and to calculate the horizontal difference, we subtract the *x*-values from each other.

Thus $rise = -3 - 1 = -4$ and $run = -1 - 1 = -2$.

$$\therefore gradient = \frac{-4}{-2} = \frac{4}{2} = 2$$

If we check back to the first example we had, the rule was
$y = 2x - 1$. Here the gradient is 2 as we calculated!!

NOTE:

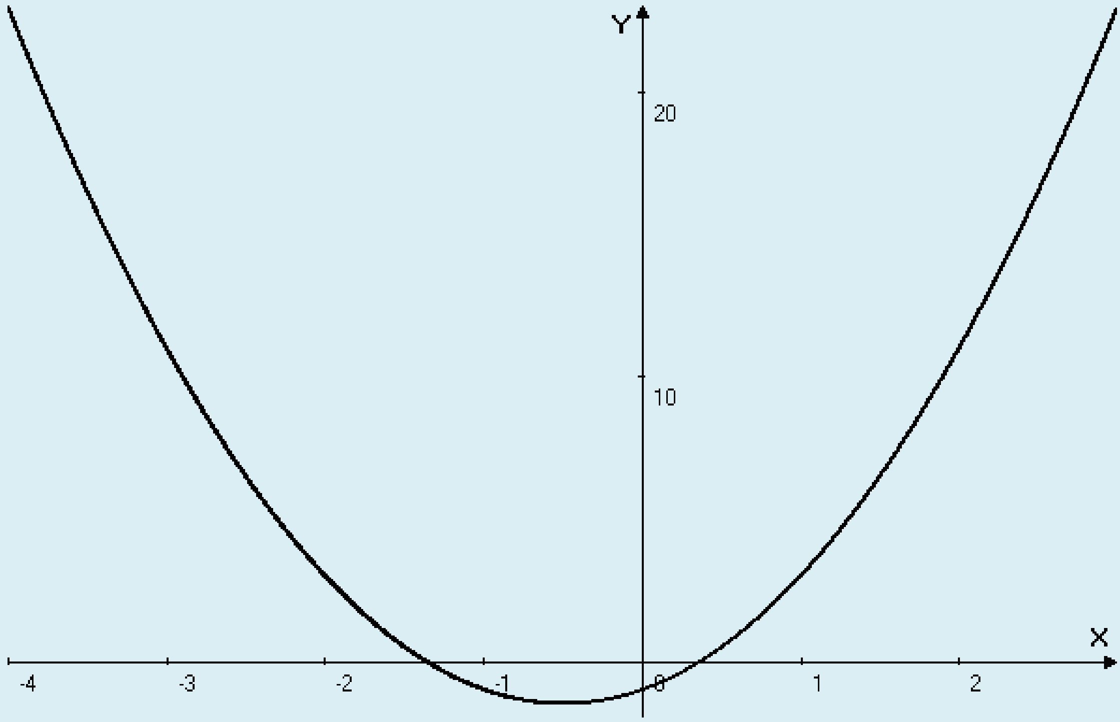
Which ever point is taken first for the vertical distance, the SAME point must be taken for the horizontal distance!!

Not all graphs are straight lines:

1) $y = 2x^2 + 2x - 1$ for $-4 \leq x \leq 3$

x	-4	-3	-2	-1	0	1	2	3
$2x^2$	32	18	8	2	0	2	8	18
$2x$	-8	-6	-4	-2	0	2	4	6
-1	-1	-1	-1	-1	-1	-1	-1	-1

$y = 2x^2 + 2x - 1$	23	11	3	-1	-1	3	11	23



<u>Activity</u>

Graphing – Ex 1 pg 212-213 and Ex 6-7 pg 218-221